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II. Solution by S. G. BARTON, Ph. D., Clarkson School of Technology, Potsdam, N. Y.

Assuming that the equation has real roots, *i. e.*, that $p^2>4q$, find Sturm's functions. They are,

$$\left.\begin{array}{l} f(x)=x^2+px+q \\ f'(x)=2x+p \\ f_2(x)=p^2-4q \end{array}\right\}$$

For $+1$, we have, $f(x)=1+p+q$, $f_1(x)=2+p$, $f_2(x)=p^2-4q$.

For -1, we have, $f(x)=1-p+q$, $f_1(x)=-2+p$, $f_2(x)=p^2-4q$.

If there is no root between $+1$ and -1, there will be the same number of variations of signs in each of these. p^2-4q is always positive. We see that the number of variations will be the same if $f(1)$ and $f(-1)$, $f'(1)$ and $f'(-1)$ have the same sign, *i. e.*, $(1+q)^2-p^2>0$ and $p^2-4>0$, *i. e.*,

$$(1+q)^2>p^2>4.$$

They will also have the same number of variations if $f'(1)$ and $f'(-1)$ have opposite signs and $f(1)$ and $f(-1)$ are both negative, *i. e.*, $p^2<4$, $1+q\pm p<0$, or $p\pm q>1$.

Also solved by G. B. M. Zerr, V. M. Spunar, and G. W. Hartwell.

309. Proposed by PROFESSOR E. B. ESCOTT, Ann Arbor, Mich.

Solve,
$$\begin{aligned} &bx^2+cy^2+az^2=ba^2+cb^2+ac^2, \\ &cx^2+ay^2+bz^2=ab^2+bc^2+ca^2, \\ &xyz=abc. \end{aligned}$$

Solution by the PROPOSER.

We see by inspection that $x=a$, $y=b$, $z=c$ is one set of solutions.

Then, put $x^2=a^2+u$, $y^2=b^2+v$, $z^2=c^2+w$. The first two equations become $bu+cv+aw=0$, $cu+av+bw=0$. Solving these, we have

$$\frac{u}{a^2-bc}=\frac{v}{b^2-ca}=\frac{w}{c^2-ab}.$$

Put these equal to s, and substitute

$$\begin{aligned} &u=(a^2-bc)s, \\ &v=(b^2-ca)s, \\ &w=(c^2-ab)s, \end{aligned}$$

in values of x^2, y^2, and z^2:

$$\begin{aligned} &x^2=a^2+(a^2-bc)s, \\ &y^2=b^2+(b^2-ca)s, \\ &z^2=c^2+(c^2-ab)s. \end{aligned}$$

Substituting these values in the third of the given equations,

$$[s(a^2-bc)+a^2][s(b^2-ca)+b^2][s(c^2-ab)+c^2]=a^2b^2c^2.$$

Two of the roots of this equation are evidently $s_1=0$, $s_2=-1$. The third root can be found by subtracting the sum of these two from the sum of the roots of the equation, which is

$$\frac{a^2}{a^2-bc}+\frac{b^2}{b^2-ca}+\frac{c^2}{c^2-ab}.$$

This gives $s_3=\dfrac{a^3b^3+b^3c^3+c^3a^3-3a^2b^2c^2}{(a^2-bc)(b^2-ca)(c^2-ab)}$.

This gives the following solutions:

$$\begin{array}{l|l|l} x=\pm a & \pm\sqrt{bc} & \pm\sqrt{[a^2+(a^2-bc)s_3]} \\ y=\pm b & \pm\sqrt{ca} & \pm\sqrt{[b^2+(b^2-ca)s_3]} \\ z=\pm c & \pm\sqrt{ab} & \pm\sqrt{[c^2+(c^2-ab)s_3]} \end{array}$$

where s_3 has the value fust found.

The signs are to be taken so that xyz is positive.

Also solved by G. W, Hartwell, G. B. M. Zerr, V. M. Spunar, J. Scheffer, J. W. Clawson, and A, H. Holmes.

PROBLEMS FOR SOLUTION.

ALGEBRA.

313. Proposed by W. J. GREENTTREET, M. A., Stroud, England.

Find the conditions that the equations $px^2+qx+r=0$ and $\alpha x^2+\beta x+\gamma+y(ax^2+bx+c)=0$ may give equal values for y.

314. Proposed by R. D. CARMICHAEL, Anniston, Ala.

Sum to infinity the series $\dfrac{1}{2.3.3.4}+\dfrac{1}{4.5.5.6}+\dfrac{1}{6.7.7.8}+\dfrac{1}{8.9.9.10}+\ldots$

315. Proposed by PROFESSOR B. F. YANNEY, Mount Union College, Alliance, Ohio.

Simplify, $1-(2-(3-\ldots-(n-1)-n)\ldots)))$.

316. Proposed by B. F. FINKEL, Ph. D.

Prove that $\sum\limits_{r=1}^{r=n}(-1)^{n-1}\dfrac{1}{n}{}_nC_r=\dfrac{1}{1}+\dfrac{1}{2}+\dfrac{1}{3}+\ldots+\dfrac{1}{n}$ where ${}_nC_r=\dfrac{n(n-1)\ldots.(n-r+1)}{1.2.3\ldots r}$. Dickson's *College Algebra,* ex. 13, p. 92.